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ARTICLE

Novel Even Beam Splitters Based on Subwavelength Binary Simple Periodic Rectangular Structure

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Abstract In this paper, a novel method of a subwavelength binary simple periodic rectangular structure is presented to realize even beam splitting by combining the rigorous couple-wave analysis with the genetic algorithm. Several even splitters in the terahertz region were designed and one of the silicon-based beam splitters designed to separate one incident beam into four emergent beams has total efficiency up to 92.23 %. Zero-order diffraction efficiency was reduced to less than 0.192 % and the error of uniformity decreased to 6.51×10^{-6}. These results break the limitation of even beam splitting based on the traditional scalar theory. In addition, the effects of the incident angle, wavelength, as well as the polarizing angle on the diffraction efficiency and uniformity were also investigated.

Keywords Beam splitters · Binary optics · Diffraction gratings · Diffraction theory · Subwavelength structures

1 Introduction

Beam splitters are widely applied in various fields such as optical communications, optical calculation, optical storage, etc. [1]. Several polarization beam splitters [2–5] and odd beam splitters for an incident beam with transverse electric (TE) polarizations have been presented, including 1 × 3 beam splitter of double-groove fused-silica gratings [6], 1 × 7 beam splitter based on double-groove blazed grating [7], three-port beam splitter of a binary fused-silica grating [8], etc. Compared with odd splitters, the isocandela even beam splitters for eliminating zero-order diffraction spectrum spot have attracted much interest due to their unique applications in circumstance with higher requirement in zero-order elimination, such as the fabrication of linearly chirped phase mask for fiber grating, beam splitting in the optical system of the lithography machine, and the denoising of the digital holographic optical system function. However, for the noise characteristic of the zero-order diffraction, the even beam splitters must achieve high efficiency and good uniformity in the condition of eliminating the noise introduced by the zero-order diffraction. The design and optimization are much more complicated and difficult than those of the odd splitters, which induces a higher requirement on the fabrication process. Many researchers have focused on binary phase grating structure which can avoid step lithography in manufacture process, e.g., Dammann gratings with complex periodic structure designed by traditional scalar theory [9–12]. The reported diffraction efficiency achieved, respectively, up to 60–83 %, and an error of uniformity down to 5.7×10^{-5}–3.2×10^{-4}. Recently, even beam splitters with a subwavelength Dammann grating based on the vector theory were proved on enhancement of the efficiency. Though the efficiency is higher than 90 %, problems existed in manufacturing process due to numerous points of sudden phase in the structure are difficult to solve, and therefore, it is hard to achieve good performance of zero-order elimination and higher efficiency. According to the traditional scalar theory, it is impossible to achieve the even beam splitter with a binary simple periodical rectangular structure because it is difficult to eliminate the zero-

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order diffraction. As far as we know, there is no report to solve this problem in spite of its importance on the increase of efficiency and uniformity.

Our group has done a lot of researches on binary optics in recent years [13–18]. Here, novel even beam splitters based on subwavelength binary simple periodic rectangular structure are presented by combining the rigorous couple-wave analysis (RCWA) with the genetic algorithm (GA). The zero-order elimination, efficiency, and uniformity as well as other properties of as-prepared splitters in terahertz region were investigated.

2 Design Theory and Method of Even Splitters Based on Subwavelength Binary Simple Periodic Rectangular Structure

Figure 1 illustrates a subwavelength binary simple periodic rectangular structure in which the refractive index distributing uniformly on the *y* axis and periodically on the *x* axis. *a* and *d* are, respectively, the ridge and the period, *h* is the depth, *f* is the duty cycle ($f = a/d$), and *H* is the thickness of the substrate. On the *z* axis, the grating area can be divided into four levels: (i) the area of $z < 0$ is the incident medium with its refractive index equal to n_1; (ii) the area of $0 < z < h$ is composed of two different mediums with their refractive index equal to n_1 and n_2, respectively; (iii) the area of $h < z < h + H$ is the substrate medium with its refractive index equal to n_2; (iv) the area of $z > H$ is the output medium with its refractive index equal to n_1. The incident medium and the output medium can be regarded as infinite, relative to that of the grating area.

A TE-polarized plane wave of wavelength λ_0 is incident upon the subwavelength structure with an angle θ and separated into *N* beams, while $N = 2L, L = 1, 2, \ldots, n$, and *n* is the natural number. When the plane wave is normally incident and $(L\lambda_0/n_1) < d < (L + 1)\lambda_0/n_1$, the grating generates *L* diffraction orders [19, 20]. Nearly all the energy is focused on the expected splitting orders, including the zero-order

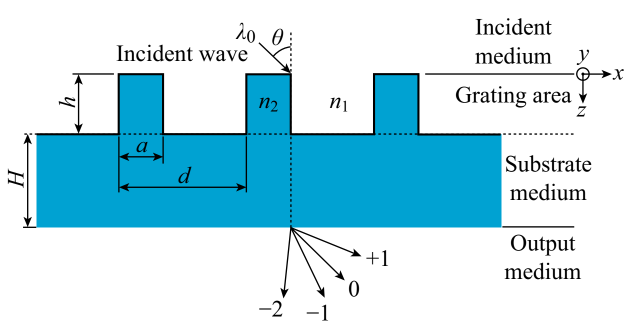

Fig. 1 Model of a subwavelength binary simple periodic rectangular structure

diffraction. By optimizing the parameters of the structure and utilizing the unique characteristic of the subwavelength structure, the guided mode resonance effect of the zero-order diffraction could be realized to form an evanescent field in the surface of the splitter. Thus, the zero-order diffraction could not transmit, leading to more energy distributed to non-zero-diffraction orders. Then a zero-order-elimination even splitter with high efficiency and low error of uniformity could be obtained. In addition, due to the strong coupling effect introduced by the subwavelength structure to the incident electromagnetic waves, there is no analytical relationship between the structure parameters and the diffraction efficiency of each order. A minor change to any of the structure parameters and the incident parameters can lead to serious degradation of zero-order-suppression, efficiency and uniformity. By varying the parameters step-by-step to obtain the optimize values is a time-consuming, low efficient and aimless process, which indicates an infeasible method. A novel method is presented in this paper to achieve even beam splitter based on a subwavelength binary simple periodic rectangular structure by combining the RCWA and the GA. A scientific and rational evaluation function was established and the optimal values of the structure parameters were obtained.

According to the RCWA, first the parsing expressions of electromagnetic field in both the incident medium and the output medium were derived from the Maxwell equations and Rayleigh expansion. By operating the Fourier expansion to the dielectric constant and electromagnetic field of the grating area as well as the substrate medium, and applying the boundary conditions of electromagnetic fields for different boundaries, a series of infinite dimension couple-wave differential equations were obtained. Eigenvalue method were used to solve the equations and numerical solutions to the diffraction efficiency of each transmitted waves P_q were figured out [21], which are sensitive to the profile parameters of the grating, such as duty cycle *f*, period *d*, the grating depth *h* and the substrate thickness *H*. Therefore, P_q can be numerically solved by the structure parameters. In fact, the key to design an even beam splitter with a subwavelength structure is how to find the optimal duty cycle *f*, period *d*, the grating depth *h* and the substrate thickness *H* to approach the minimum zero-order diffraction efficiency, the maximum sum of each non-zero-order diffraction efficiency, and uniform distribution. Considering these goals above, an evaluation function can be established as follows:

$$F(f, d, h, H) = \alpha \sum_{q=-L, q\neq 0}^{L} \left| P_q - \frac{P}{2L} \right| + \beta U + \mu |1 - P| + \nu P_0, \tag{1}$$

where $2L$ is the number of splitting beams, P_q is numerically solved by the RCWA, *P* is the sum of each non-zero-order diffraction efficiency (total efficiency), and *U* is the error of uniformity defined by Expression 2 and 3 [9].

Table 1 Results of designed even beam splitters

Material/frequency		Parameters of the structure					Parameters of the performance									
Refractive index	Frequency (THz)	Period d (μm)	Grating depth h (μm)	Substrate thickness H (μm)	Duty cycle f	Minimum feature size (μm)	N	Distribution of the efficiency of each diffraction order P_q (%)							Total efficiency P (%)	Error of uniformity U (%)
								P_{-3}	P_{-2}	P_{-1}	P_0	P_{+1}	P_{+2}	P_{+3}		
3.42	2.52	149.6	71.6	125.0	0.467	69.9	2	0	0	49.50755819225	0.22424983259	49.50755819224	0	0	99.01	1.01×10^{-13}
		269.7	175.2	18.1	0.409	110.2	4	0	23.0586	23.0589	0.192	23.0589	23.0586	0	92.23	6.51×10^{-4}
		398.1	381.2	55.6	0.373	148.5	6	14.09	14.64	13.92	2.511	13.92	14.64	14.09	85.30	2.52
1.52	1	561.5	346.7	208.6	0.227	127.4	2	0	0	49.7479081315	0.238	49.7479081317	0	0	99.50	2.01×10^{-10}
		869.3	384.2	37.0	0.380	330.1	4	0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}
		933.6	999.4	647.7	0.321	299.8	6	15.789	15.787	15.797	0.572	15.797	15.797	15.789	94.75	0.032
	1.40	620.9	274.4	26.4	0.380	235.8	4	0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}
	1.63	533.3	235.7	22.7	0.380	202.5		0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}
	1.89	459.9	203.3	19.6	0.380	174.7		0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}
	2.45	354.9	156.8	15.1	0.380	134.7		0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}
	2.52	344.3	152.2	14.7	0.380	130.8		0	23.1752	23.1759	2.266	23.1759	23.1752	0	92.70	1.51×10^{-3}

$$P = \sum_{q=-L, q\neq 0}^{L} P_q, \quad (2)$$

$$U = (P_{\max} - P_{\min})/(P_{\max} + P_{\min}), \quad (3)$$

where $P_{\max}$ and $P_{\min}$ are, respectively, the maximum and minimum diffraction efficiency of the non-zero-order. The uniformity, the total diffraction efficiency and the suppression of the zero-order are, respectively, considered in each term in Expression 1. α, β, μ, and v are the weighting factor ranging from 0 to 1, with their total equaling to 1, which can be set when necessary. Therefore, the nature of the design is to find a set of optimal parameters of the rectangular periodic grating in order to get a minimum evaluated function F. However, P_q is a multi-variables function and a numerical solution, which could not be explicitly expressed as a function of the parameters above. With the principle of jumping out of local extreme point, GA is especially suitable for solving multi-variable and discrete variable optimization problems [22, 23]. Therefore, GA was applied to optimize the evaluation function F. First, initial values of the f, d, h, and H were set and the initial population was produced. Second, the efficiency of each diffraction order as well as the evaluation function F is calculated. Third, if the convergence conditions are not satisfied, selection, intersection, and mutation were done to the population. Then the evaluation function F is recalculated and the convergence conditions are rejudged. The optimized parameters of f, d, h, and H are output until the convergence conditions are satisfied. Then the optimal parameters of the splitter can be obtained.

3 Optimization Example and Results

With the optimize method above, several even beam splitters with subwavelength binary rectangular periodic structure were designed to realize 2, 4, and 6 emergent beams. The initial conditions were set as below: A TE-polarized terahertz plane wave, with its frequency, respectively, equal to 1, 1.40, 1.63, 1.89, 2.45, and 2.52 THz, is normally incident upon the beam splitter. The incident medium is air with its refractive index $n_1 = 1.0$, and the material of the splitter is either silicon with its refractive index $n_2 = 3.42$ or polyethylene with its refractive index $n_2 = 1.52$. The optimal results are denoted in Table 1. For even splitters with their numbers of splitting beams 2, 4, and 6, diffraction efficiency of each output order are shown in Fig. 2.

In Fig. 2, zero-order elimination, high efficiency and uniform beam splitting are effectively realized. The highest total efficiency reaches to 99.50 % and the error of

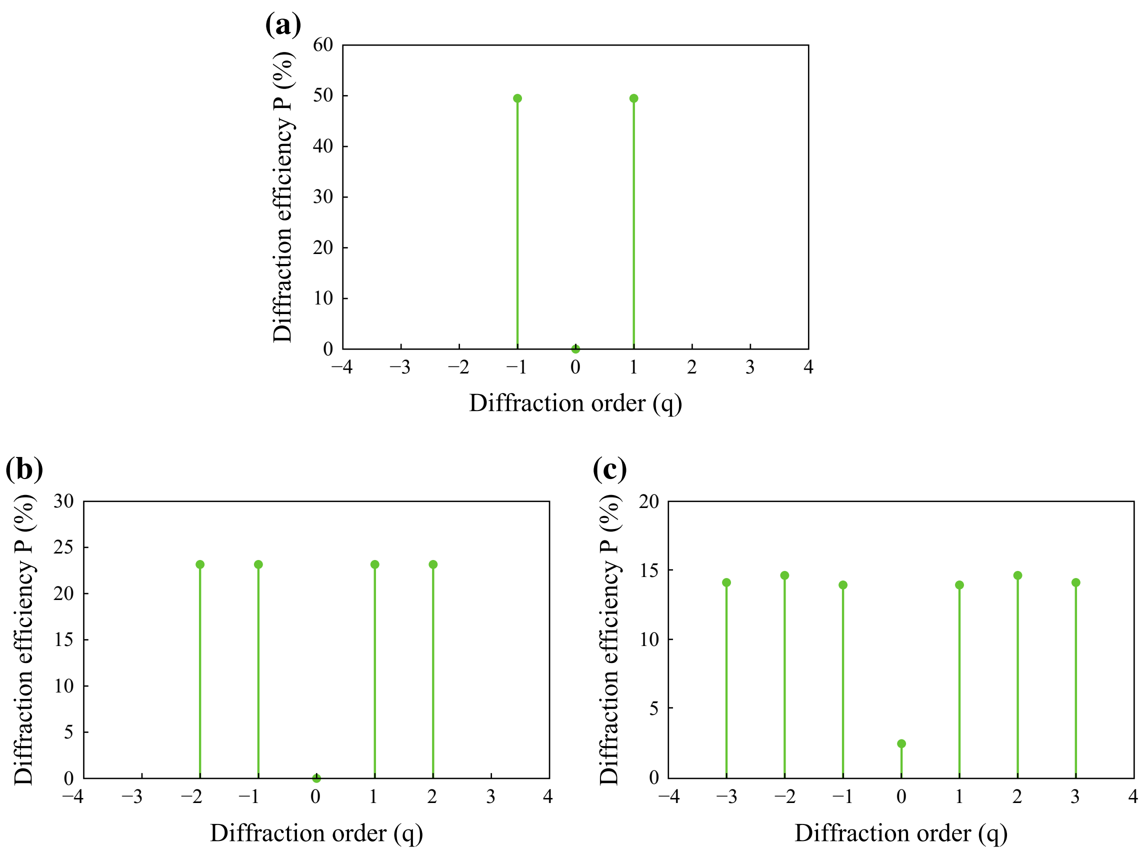

Fig. 2 Distributions of the efficiency of each diffraction order of the silicon-based even splitters. **a** Silicon-based 2-beam splitter. **b** Silicon-based 4-beam splitter. **c** Silicon-based 6-beam splitter

uniformity is better than 2.01×10^{-12}. One of the designed silicon-based four-beam splitters has total efficiency up to 92.23 % with a preferable result of reducing zero-order diffraction efficiency to 0.192 % and an error of uniformity down to 6.51×10^{-6}, which indicates that the uniform distribution of energy for each beam is implemented. Similar results can be obtained while using polyethylene as the substrate.

4 Influence of Incident Parameters on the Performance of the Beam Splitter

The law of influence of incident parameters (λ, Ψ, θ) on the diffraction efficiency and its uniformity is exposed as shown in Figs. 3, 4, 5.

According to the grating theories, diffraction efficiency of each non-zero orders is symmetrically distributed when concerning the zero-order. So only P_0, P_{+1}, and P_{+2} need to be analyzed, as denoted in curves T_0, T_1, and T_2 in Fig. 3. Obviously, U decreases and P_0 increases with a deviation of λ from its designed value. An error of uniformity lower than 18.81 % and total diffraction efficiency higher than 71.69 % were achieved within a 1-μm incident wavelength bandwidth.

Figure 4 shows that when polarizing angle $\Psi = 90°$, i.e., TE-polarized, diffraction efficiency of each non-zero order are equal and P_0 is near 0, which indicates a high uniformity and total diffraction efficiency. With the

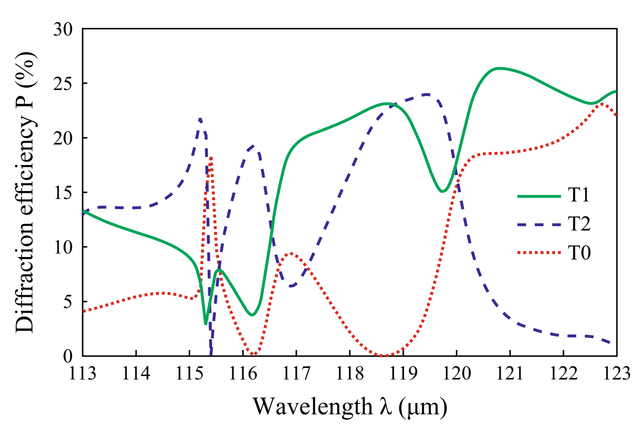

Fig. 3 Diffraction efficiency of each order versus the incident wavelength λ

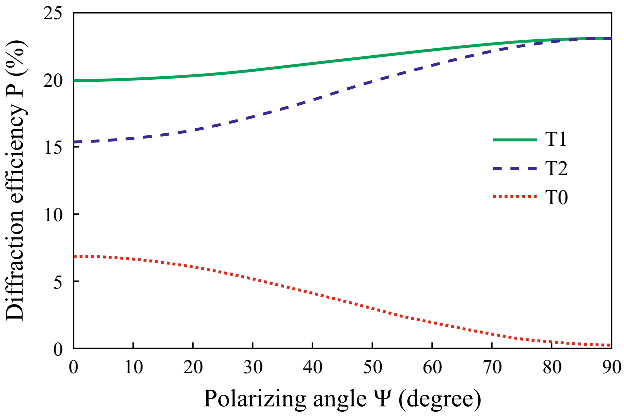

Fig. 4 Diffraction efficiency of each order versus the polarizing angle Ψ

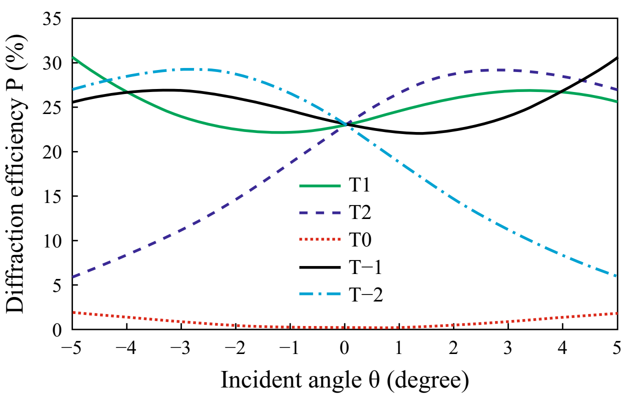

Fig. 5 Diffraction efficiency of each order versus incident angle θ

decrease of Ψ, $P_{\pm 2}$ decreases, while $P_{\pm 1}$ is little changed, and P_0 increases, which indicates a degradation of the uniformity and zero-order suppression. When $\Psi = 0°$, i.e., TM-polarized, the uniformity, diffraction efficiency and zero-order suppression effect achieve at their worst level. An error of uniformity lower than 1.84 % and total diffraction efficiency higher than 88.35 % are achieved with a deviation to the polarizing angle less than 25°.

Figure 5 shows that variation of θ has a modest impact on the suppression of P_0 but a great impact on the efficiency distribution of the non-zero order discussed above, leading to $P_{+q} \neq P_{-q}$ ($q \neq 0$), which degrades the uniformity. An error of uniformity lower than 17.34 % and a total diffraction efficiency higher than 92.21 % were achieved with a deviation to the incident angle less than 1°.

In conclusion, a relatively good uniformity and high diffraction efficiency of the element can be maintained with a deviation of wavelength less than 1 µm, a deviation to the polarizing angle less than 25°, and a deviation to the incident angle less than 1°.

5 Conclusion

In this paper, a novel method is presented to realize even beam splitting based on a subwavelength binary simple periodic rectangular structure. Even beam splitters in the terahertz region were designed. The highest total efficiency reaches to 99.50 % and the error of uniformity is better than 2.01×10^{-12}. One of designed silicon-based four-beam splitters has total efficiency up to 92.23 % with a preferable result of reducing zero-order diffraction efficiency to 0.192 % and an error of uniformity down to 6.51×10^{-6}, which indicates that the uniform distribution of energy for each beam is implemented. Taking the silicon-based four-beam splitter for example, the influence of the incident wavelength, the polarizing angle and the incident angle on the diffraction efficiency and its uniformity were investigated. It was found that the total diffraction efficiency and the uniformity were sensitive to the incident wavelength, that is, with reduction of the polarizing angle the total diffraction efficiency decreased gradually, whereas, the uniformity degraded. In addition, the incident angle was found to have a modest impact on the suppression of P_0, whereas, it has a great impact on the efficiency distribution of the non-zero order. This will lead to a great change on the uniformity. It can be said that we presented a novel method to solve the problems of zero-order elimination, diffraction efficiency and uniformity improvements of even beam splitting by applying the binary simple periodical rectangular structure. Our results could be expected to apply in fabrication of linearly chirped phase mask for fiber grating, beam splitting in the optical system of the lithography machine, and the denoising of the digital holographic optical system function.

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